Formulas: Inherent Availability and Reliability with Constant Failure and Repair Rates[1]

	At a particular point in time (t)	**During the time interval** $[t_1, t_2]$	**During the time interval** $[t_1, t_2]$ when $t_2 \rightarrow \infty$
Inherent Availability[2]	$A(t)_{sys} = \left[\frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t} \right]^N$ This is "point availability."	$A(t_1,t_2)_{sys} = \left[\frac{\mu}{\lambda+\mu} + \frac{\lambda}{(\lambda+\mu)^2 (t_2-t_1)} [e^{-(\lambda+\mu)t_1} - e^{-(\lambda+\mu)t_2}] \right]^N$ This is point availability averaged over the time interval from t_1 to t_2.	$A_{inherent-sys} = \left[\frac{\mu}{\lambda+\mu} \right]^N = \left[\frac{MTBF}{MTBF+MTTR} \right]^N$
Reliability[3] (Availability = Reliability when μ = 0)		$R(t_2 \mid t_1)_{sys} = \left[e^{-(\lambda)(\Delta t)} \right]^N = e^{-N\lambda\Delta t}$ where $\Delta t = t_2 - t_1$ or $R(t)_{sys} = \left[e^{-\lambda t} \right]^N = e^{-N\lambda t}$ when $t_1 = 0$ and $t_2 = t$. This is true because the distribution is exponential.	$R(t_2 \mid t_1) \rightarrow 0$ as $t_2 \rightarrow \infty$
Probability of r or less events[4]		$P = \sum_{n=0}^{r} \frac{e^{-N\lambda t}(N\lambda t)^n}{n!}$ where t is from 0 to t_2.	$P \rightarrow 0$ as $t_2 \rightarrow \infty$
Probability of element cycle time is more than c time units[5]		$G(c) = \frac{\mu e^{-\lambda c} - \lambda e^{-\mu c}}{\mu - \lambda}$ where c is the lower bound for predicted time for element operation, failure, and repair.	$G(c) \rightarrow 0$ as $c \rightarrow \infty$

Notes:

1 - Nomenclature: **N** is the number of elements in a series configuration; **MTBF** is element mean time between failure; **MTTR** is element mean time to repair; **λ** = 1/MTBF; **μ** = 1/MTTR; **t** is mission time; and **c** is cycle time. Notes: λ and μ are constant rates over time (thus, the exponential distribution models the failure and repair distributions). λ, μ, t, and c have the same unit of time (e.g., hours). For design purposes, MTBF is a lower-bound parameter and MTTR is an upper-bound parameter.

2 - **A** is Availability, the probability of mission readiness at a particular point in time or during a time interval from t_1 to t_2. Steady-State Availability is asymptotic as $t \rightarrow \infty$. Steady-State-Inherent Availability occurs at the 6th decimal place when $(\lambda+\mu)t$ is approximately 10 or more.

3 - **R** is Reliability, the probability of no failures during a time interval from t_1 to t_2. R is 0.3679 or less when $N\lambda t$ is 1.0 or more. The notation $R(t_2 \mid t_1)$, a conditional probability, means the "reliability from t_1 to t_2 given the system has operated during the time interval from 0 to t_1 with no failures."

4 - **P** is the probability of r or less number of events (e.g., failures, repairs, etc) during the time period from 0 to t. This probability is determined by the cumulative Poisson distribution. The Poisson process assumes failures are immediately repaired or replaced—thus, there is no accounting for repair time.

5 - **G** is the probability that element (not system) cycle time is more than c time units in length. Cycle time, a convolution, accounts for element operation, failure, and repair.

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